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October 9.

Magnetic Φ,

I gave you very great credit for finding out the objections to my theory of quadratic displacements. Townsend at first denied the truth of my $=^{n}$ of the bent beam, but he now acknowledges that it is right. He does not agree with my direction of the pressure, and he sent me his value which I promptly attacked as founded on error. I am almost completely satisfied that the whole of my solution of the problem is ~~correct~~ correct, & I sent my method to Townsend. The $=^{n}$ of the beam being now by a triumvirate consent

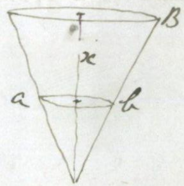
$$y = \frac{k}{h}x - \frac{Wl'^4}{24\,lh^4 EI}\left(h^3x - 2hx^3 + x^4\right),$$

we have $\frac{1}{\rho}$ at any point, and the bending m^t. $M = \frac{EI}{\rho}$. Now, Φ, let me give you a

The ends are not tangentially fixed. If they were, the problem would be radically different. Your assumption that the coefft. of δds is constant is the same as assuming that the mass per unit length is const. — as I have done. I cannot see how Lagrange would have made out that the coefft. of $\delta d\theta \propto \frac{1}{\rho}$. We know otherwise that bending moment $= \frac{EI}{\rho}$.

How is it that you have sent me no proofs. Have you not had any from the Clarendon Press? I shall write to the Superintendent of the Press to address them in future to T.C.D.

Another point. A heavy solid cone has its base fixed horiz^{ly}. and it hangs; find its elongation. I think Townsend solves it thus:



Force per unit of sectional area over ab

$= \frac{W}{\pi r^2}\frac{h-x}{h}$, and by Hooke's Law this $= E\frac{du}{dx}$, if u is disp^t. parallel to x, & E = Young's modulus.

Hence $u = \frac{W}{\pi r^2 h E}\int_0^h (h-x)\,dx$ = whole elong^n.

I am strongly disposed to quarrel with this. What we have really is $\frac{W}{\pi r^2}\frac{h-x}{h} = \lambda\theta + 2\mu\frac{du}{dx}$ (Lamé);

but who knows what the law of the v, w displacem^{ts} is? How can the formula for the extension of a cylindrical

a little bit of useful information, viz., —
The Shearing force at any normal section is equal to the diffl. coefft of the Bending Moment with regard to the arc. For, to prove this look simply at this figure:

$N = \frac{dM}{ds}$.

Calculate M, and find $\frac{dM}{ds}$ at A. This will $= R\cos\overline{\varphi+\alpha}$, where α = incl.n of tan. at A to horizon.

In this way we have

$$R\cos\overline{\varphi+\alpha} = \frac{Wl'}{2l}\left(1+\frac{Wkl'^2}{8lEI}\right)\cos\alpha$$

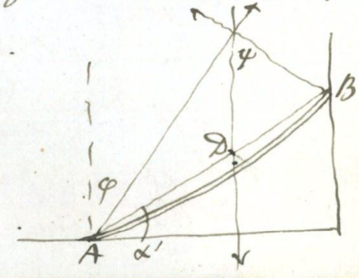
(where $l' = \sqrt{k^2+h^2}$, wh. = new length of beam to order of approx. adopted)

$$\therefore R\sin\varphi(\cot\varphi - \tan\alpha) = \frac{Wl'}{2l}\left(1+\frac{Wkl'^2}{8lEI}\right) \ldots (I)$$

Townsend made, I think, the mistake of supposing that the vertl. thro' C.G. of beam bisects it or its chord. It does not do so.

We easily find

$$\frac{AD}{DB} = 1 + \frac{Wkl'^2}{30lEI},$$



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$$\therefore \left(2+\frac{Wkl'^2}{30lEI}\right)\tan\alpha' = \left(1+\frac{Wkl'^2}{30lEI}\right)\cot\varphi - \cot\psi \ldots (II),$$

by the well known cotangent formula.

Also, by Δ of forces,

$$R\sin\varphi(\cot\varphi + \cot\psi) = W \quad \ldots (III)$$

Divide (III) by (I) and combine with (II) to eliminate ψ, remembering that

$$\tan\alpha = \frac{k}{h} - \frac{Wl'^4}{24lhEI},$$

and I am a Dutchman if you don't get

$$\cot\varphi = \frac{k}{h} + \frac{Wl'^4}{24hEI\Delta l}$$

Where $\Delta l = l' - l$.

I confess that in getting $\frac{AD}{DB}$ by finding the x of C.G., I have assumed the beam to be uniform; otherwise $\bar{x}$ is not $= \frac{\int x\,ds}{\int ds}$.

If we had to allow for variation of mass of unit length, there would be 7 devils at least to pay.

I think that R will be found $= W\frac{k}{2l'} + \frac{12EI\Delta l}{l'^3}$, (in wh. l may be used for l').

The method of Lagrange was the first that I tried, but I could make nothing of it.