

Paington Devon

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Dear Fitzgerald,

14/2
A man who makes himself disagreeable as a would-be reformer gets more kicks than halfpence, I find, as a general rule, so I am glad you like my treatment of differentials, at any rate. I can hardly expect any quaternionist to like the notation changed, for the obvious reason of habit, if there were no other. I should like, however, to make my position clear ~~absolutely~~ as regards my vector analysis. Put very briefly it is this. From the purely quaternionic pt^e of view I say that Hamilton is right, and $i^2 = -1$ with his ideas; and that Quaternions is unique - (Too unique). On the other hand I say that the quaternionic system is not the way that vector analysis presents itself in mathematical physics. Take a good Cartesian work, say Gregorj's Solid Geometry, & put it into vector algebra. The quaternion would not show itself at all. It is the same with mathematical physics generally. There is no quaternion in it, and there is none when put into vector form, (that is, supposing you don't know quaternions).

Now Quaternions is so difficult that it is I think hopeless to expect any general use of the system by ordinary math^m. It is very hard; it is out of harmony with scalar methods; and it is not wanted in physical mathematics (i.e. the quaternionic part).

But on the other hand, if you ignore the quaternion altogether, you at once get a simpler system, not on a quat. basis, but vectors simply. Now this is the way vectors present themselves in math^t-physics; it is comparatively quite easy to understand (by the absoluteness of the quat. and the double meaning of a vector, a translation or a rotation); and it can be made to perfectly harmonise with scalar methods. I therefore believe this system is thoroughly good for practical use, not merely by a few accomplished ^{quaternionic} math^m, but by a large body of men who need not even know what a quat. is. I have only adopted this system after the most mature consideration of the pros and cons. As regards $\underline{ab} = \underline{a}\underline{b}$ to mean scalar product of \underline{a} and \underline{b} according to

$$\underline{ab} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

if you put the quats out of sight and regard it from the point of view above mentioned, it is obviously right. Suppose you

construct a vector algebra out of Cartesians by systematic abbreviation, how would you possibly endow the function $a_1 b_1 + a_2 b_2 + a_3 b_3$ with the - sign? You would never think of such a thing. Only by the unique and out-of-the-way quaternions do you come to $i^2 = -1$.

I think my Chap III on Vec. Anal. will be useful as a stop gap, till some one writes a treatise on Vec. An. as it appears in Phys. math's, which is what I propose to exhibit. Besides that, people have got to learn to think in terms of vectors. Well, they may do so thro' a simple kind of v. analysis; but I don't think they can possibly do it in terms of quats.

I can imagine in the distant future such a great development of mental powers all round that quats. may become easy. Then let them use quats. if they want to, or something better still, perhaps. The elementary vec. an. will have served its purpose.

Poincaré has such a high reputation as a math's that I was much disappointed by what I read in Nature about his researches. Such a lot of useless and worse than useless stuff in it.

In regard to the new man Macaulay, I had no time to study his R.S. paper. But I ~~can~~ see he is a very clever math's. [By the way, on the matter of quats. again, I see he is an ultra-quaternionist, wants it to be independent of the scalar

math's (like Hamilton, and Tait). I entirely differ. & think they shd go hand in hand, in mutual harmony.] He has also something special of his own dealing with moving media. But I don't like the way he treats the subject. He overloads it with his math's. And he doesn't seem to be acquainted with other work than Maxwell's in other ways, so that there is a ~~curious~~ curious mixture of very advanced math's combined with old-fashioned electrical ideas. Then again, he hangs on too many secondary matters. It is not practical. And I don't see why he should start in the way he does, with an arbitrarily chosen imperfect scheme, only to fill up the missing links afterward by such a laborious method. You may make too much of Lagrange's eqns of $m^2 = +\infty$ forth.

Still, I can't help thinking that, with his math's talent, he is one of the coming men, from whom much may be expected?

Your remark about $V_a V_{b c}$ involves the assumption $i^2 = -1$, I think. In my notation

$V_a V_{b c} = \cancel{b.c.a.} - c.a.b$
by defn of a scalar & a v. product. If $a = b$ = unit vector $\overset{i \text{ say}}{\wedge}$ and $a \perp c$ also, it reduces to

$$V_i V_{i c} = -c \quad \text{when } i^2 = 0$$

Yours sincerely

Olin Levi Warner.