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My dear Dr FitzGerald.

I am afraid my last letter was rather diffuse. Dr Schuster tells me that we shall have you over here soon, when I hope I may be able to present my difficulties more clearly.

My experiments, on the relation between absorption & fluorescence are at length working satisfactorily & seem to show a marked difference in the former, for the rays of 'true dispersion' where fluorescence takes place.

I am not ~~certain~~ clear what the absorption would be in this case. The calculation of the diff in the power of the particle when vibrating under the influence of the forces & when vibrating harmonically under the force $(-p^2)$ becomes very complicated.

In the case of phosphorescent bodies Becquerel rejected the force $\gamma\left(\frac{du}{dt}\right)$ as it led to the law $i = i_0 e^{-\kappa t}$ for the extinction of the phosphorescent light which he found not to be true & used a force $\pm\gamma\left(\frac{dx}{dt}\right)^2$. From which he deduces results which appear to agree with exp. I think that if we regard the ether as vibrating thus $U = A\cos pt$ U being its displacement at any instant in space & if y = that of molecule. we have then

$$\frac{d^2y}{dt^2} + \kappa y \pm \gamma\left[\frac{d}{dt}(U-y)\right]^2 = 0$$

Let $v = U - y$ & this reduces to

$$\frac{d^2v}{dt^2} + \alpha v + \beta\left(\frac{dv}{dt}\right)^2 = A\cos pt.$$

I can get $\left(\frac{dv}{dt}\right)^2 = Ae^{\lambda v} + Bv + C$ for the left hand side = 0. ~~& then expand in series~~ but I don't see how we shall get any farther.

The question about whether the absorption would depend upon the amplitude of the vibrating molecules [or I suppose more properly the vibrating electrons,] is a difficult one! ~~Thus Thus~~

You say if an alternating E.M.F. $e = e_0 \cos pt$ doing work on an alternating current $c = c_0 \cos pt$, it does work at the rate $= \frac{1}{2} e_0 c_0 =$ power $\therefore$ power absorbed is greater the greater c_0.

I investigated the question in this manner. If a particle vibrating harmonically be acted upon by a force which varies harmonically & if the two are in the same phase we have

$$\frac{d^2y}{dt^2} + p^2 y = f_0 \cos pt.$$

which gives

$$\frac{dy}{dt} = At.\cos pt + B\sin pt$$

$$y = \frac{At}{p}\sin pt + y_0 \cos pt$$

where y_0 is the max ~~at~~ displacement of particle

Power ~~of~~ which vibrating particle absorbs (or its increase in its mean energy) ~~[struck out]~~

$$= \frac{m}{T^2}\int_0^T \frac{1}{2}\left(\frac{dy}{dt}\right)^2 . dt . - \frac{m}{4}.\frac{y_0^2 p^2}{T}$$

which comes out to be $= \frac{m}{4}\left[\frac{f_0^2}{4}.\left(\frac{T}{3} + \frac{1}{4p^2 T}\right) - \frac{f_0 y_0}{T}\right]$

but $p = \frac{2\pi}{T}$

$$= \frac{m}{4}\left[\frac{f_0^2}{4}\left(\frac{1}{3} + \frac{1}{4\pi^2}\right).T - \frac{f_0 y_0}{T}\right]$$

$$= \left[\alpha . T - \beta . \frac{y_0}{T}.\right]$$

So that accordingly the greater y_0 the less the absorption. If this be so then bodies at a high temperature ought to ~~be~~ absorb less than at low temperatures, which might account for some bodies becoming transparent ~~in~~ in the course of being heated.

Schuster thinks a ~~term~~ force such as $K\left(\frac{dy}{dt}\right)$ should be introduced into the equation. In which case writing the eq. thus

$$\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + p^2 y = f_0 \cos pt$$

we get

$$y = e^{\alpha}\left[Ae^{+\beta t} + Be^{-\beta t}\right] + \frac{f_0}{\gamma p}\sin pt.$$

where $\beta = -\frac{\gamma}{2} + \frac{\sqrt{\gamma^2 - 4p^2}}{2}$

$\alpha = -\frac{\gamma}{2}$

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