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Palazzo Ruffo.
Via Amedeo
Napoli
Oct. 12. 1888

To Professor Fitzgerald
Trinity Coll: Dublin.

Dear Sir,

You may perhaps remember my name when I remind you of the paper I read on "A fourth Dimension" at this years meeting of the British Association. I venture to write to you now on that subject as I find that any effect that that paper might have had on the minds of those who heard it was practically nullified by the remarks you made after it, to the effect that "time" might in the same way be taken to represent a fifth dimension. This result was I believe due not only to the authority with which of course you speak on such a subject, but also to the fact that you had the last word in the argument, I having had no opportunity of replying. I hope therefore that you will pardon my trespassing on your valuable time, and if it is not too much to ask, that you will clear up the following points in the discussion.

One argument I adduced in favour of considering 'density' or rather as I prefer to call it 'solidity' as

the fourth dimension, was that it enabled us to represent equations of four independent variables graphically. The only argument you adduced in favour of considering time as a fifth was that with its aid it was possible to represent equations of five variables. I presume you had something of this sort in your mind. The distribution of heat radiating by conduction from a source of heat in a solid body is, at any moment, a function of four variables, namely the three ordinary coordinates of any point and the temperature at that point – which is an example of my fourth dimension 'solidity'. But as the time is also an independent variable the distribution considered generally may be called a function of five variables. But I cannot see how you can call time a dimension of the distribution. The sort of the distribution namely the total amount of heat, is altered if the temperature of the parts is altered, but in what sense can you say it is affected by any alteration of time? To the series length, area, volume, I have added a fourth term, mathematical mass, in this case amount of heat. What fifth term do you add if you call time a fifth dimension?

You may reply ~~that~~ by admitting that time is not in every respect analogous to a dimension, but that no more is 'solidity', even though more closely analogous in some respects. Perhaps what I am writing will seem to you merely special pleading – an attempt to bolster up a fantastic notion. But I have tried all I could – especially since the meeting at Bath – to look upon my idea as a purely

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fantastic one, but in vain. To me indeed geometry of three dimensions seems only a special case, a projection out of geometry of four, just as plane geometry is only a special case of geometry of three dimensions. ~~For~~ For, there exist, in common life, geometrical shapes which can <u>not be represented</u> in three dimensions You will of course admit that the function of geometry is to define the shapes of bodies. You will scarcely deny that such a body as Professor Everitt not only discussed, but actually <u>attempted to draw on the black board</u>, namely a "medium of varying density", has a shape. The earths atmosphere may be taken as a case in point. If, then, geometry has no more than three dimensions, can you define such a shape in three dimensions? Or, Can you give me an approximate equation to the shape of the earths atmosphere in three independent variables? Professor Everitt represented his medium on the black board by a series of lines, which were sections of surfaces of constant density. These surfaces correspond <u>exactly</u> to the 'contours' by which hills are represented upon a map. To deny the existance of a fourth dimension in geometry because the shape of a medium of varying density can be ~~represented~~ sufficiently indicated in three, by 'surfaces of constant density' is as absurd as to deny the existance of a third dimension because hills can be represented on a map by contours.

I must however emphasise the fact that 'solidity' and 'density' are not necessarily the same thing. Any differentiable

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from surrounding space may be called 'solidity'. I have already mentioned an example in which solidity was represented by temperature. In problems about 'equipotential surfaces' it is represented by electrical potential. But it is quite impossible to conceive a shape in which there is no such differentiation whatever. If you consider the shape of a portion of space not differentiated in any way from the rest of space, but marked off by a boundary only, (this is all that is ever done in what is vulgarly called 'solid geometry') you are really considering not the shape of a solid at all, but only of a bounding surface. It is however still true that this surface must have solidity, though it may be so small as to be practically negligeable, just as its thickness cannot be absolutely zero, though it may be as small as you please. If there were no differentiation between the bounding film and space on either side of it, it would be impossible to tell where the film was, or to define its shape.

The only formidable objection to geometry of four dimensions is the difficulty of actually conceiving a fourth independent direction; which difficulty I have however overcome in my own mind, and have no doubt is not insurmountable in the minds of others. The difficulty arises solely from the fact that physical matter can only be moved in three independent directions, and consequently we have not the daily practice which has enabled us to realise ~~the other~~ three independent directions. But in

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imagination it is possible to conceive a shape in three dimensions, and then, as it were, take a peep round the corner to realise the fourth, just as we see a picture in two dimensions but require also a 'side elevation' to realise the third. However this part of the subject is really psychology and not mathematics, so I will not bore you further with my views upon it.

Hoping that you will find time shortly to read and reply to this memoir, I remain

Yours sincerely

Edward. T. Dixon

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