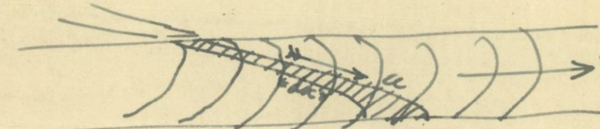
32, EGLANTINE AVENUE,
BELFAST.

Nov 2 1896

Dear George

The way the turbine equation comes about is this



T is velocity of turbine itself.
u " " " fluid
θ " angle of flow to direction of T
x " distance along path shaded

Then, if there be no heat supply, and $\frac{1}{V}$ be the density we have

$0 =$ (change of internal heat, dI)
$+$ (work of volume-increase, PdV)
$+$ (work pushing a block of fluid along against dP, VdP, being $\frac{dP}{dx}dx.a.u\,dt$, and $au = V$)
$+$ (change of kinetic energy, $u\,du$)
$+$ (work of turbine at block dx)

nozzle) $u^2 < \gamma PV$ so long as the pressure is falling; and $u^2 = \gamma PV$ at the neck, which, as I said, works out correct results, which differ totally from what one might be tempted to conclude by direct integration and saying P at outlet will be external pressure

$\therefore u^2/2 = \frac{\gamma}{\gamma-1}(P_0V_0 - P_EV)$, the the true result being $u^2/2 = \frac{\gamma}{\gamma-1}(P_0V_0 - P_1V_1)$ where $P_1V_1 = \frac{2}{\gamma+1}P_0V_0$. I was looking to see if there was not a trap of the same sort in the turbine part of the equation also, arising in the same way — viz: from the pressure being still falling where $\frac{du}{dx} = 0$.

Your affectionate brother
Maurice F. FitzGerald

Love to Harriette and the children
All well here, except Mrs Charnock who has had a bad cold, but is better.

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Now to get this last, we have momentum entering block $dx\, d\theta$ in direction of T is $u\cos\theta$, and leaving it is $u\cos\theta + dx.\frac{d}{dx}(u\cos\theta)$, per unit discharged per unit of time, whose <u>loss</u> of momentum is ∴ $-dx\frac{d}{dx}(u\cos\theta)$ which is ∴ (treating the vanes as infinitely thin and numerous) the forward force on the turbine, whose velocity is T, and its work is therefore $-T dt.\, dx.\frac{d}{dx}(u\cos\theta)$ in time dt. These all tot up to

$$\frac{\gamma}{\gamma-1}\frac{d}{dx}(PV) + u\frac{du}{dx} - T\frac{d}{dx}(u\cos\theta) = 0$$

If $T=0$ (your curved tube case) this reduces to (since $\frac{\gamma}{\gamma-1}dPV = VdP = -\gamma P dV$) to $V\frac{dP}{dx} = -u\frac{du}{dx}$ for the tube at rest and for a turbine worked by a liquid when $V = \frac{1}{\rho}$ $\frac{dP}{dx} = \rho\frac{dH}{dx}$ becomes

$$\frac{dH}{dx} + u\frac{du}{dx} = T\frac{d}{dx}(u\cos\theta)$$

which are right I think. Anyway my theory doesn't make out that it makes no difference whether we use

$$\frac{\gamma}{\gamma-1}\frac{d}{dx}(PV) + u\frac{du}{dx} - T\frac{d}{dx}(u\cos\theta) = 0$$

with T = something or nothing, <u>except</u> at a place where $\frac{d}{dx}(u\cos\theta) = 0$ independently, by θ being $\pi/2$ and $\frac{d\theta}{dx} = 0$, severally.

With the tube at rest we have (putting $V\frac{dP}{dx} = -\gamma P\frac{dV}{dx}$, and $au = V$, on writing out and transposing, and multiplying across by V)

$$(u^2 - \gamma PV)\frac{dV}{dx} = u^3\frac{da}{dx}$$

and the corresponding equation, which was the one I gave you, if the T term be left in for the turbine case. In the tube at rest, if $\frac{da}{dx}$ be negative (i.e. a contracting